VII. PLAUSIBLE REASONING

In this chapter the foundations for a structure of Plausible Reasoning are explored. By the word *plausible* is meant the kind of reasoning that is persuasive without being formal. It is the kind of reasoning most often used in everyday thinking, particularly in critical thinking. While plausible reasoning is in fact based upon formal logic this relationship is not readily apparent. Plausible reasoning is to be found in courts of law, in political campaign rhetoric, in the doctor's office, in an auto dealer's showroom, in history books, on a therapist's couch — in fact almost everywhere that people attempt to persuade others to their points of view, draw conclusions from observations, invent new social institutions, or make speculations of one kind or another. The "father" of plausible reasoning is Georg Polya who wrote the seminal works in the field, the two volume set *Mathematics and Plausible Reasoning.* He referred to his later work *How to Solve It* as a dictionary of problem solving heuristic. As in Chapter II, the name of Thomas Bayes is featured in discussions of plausible reasoning because such reasoning is based primarily upon subjective probability. It is what people generally use in communicating ideas, exchanging views, and refining both their questions and their conclusions.

To illustrate the widespread use of plausible reasoning, a Hispanic laborer was once noted to be carrying a bottle of peppermint schnapps to the checkout register. A curious person behind him in line thought this strange and asked the man why he was purchasing a bottle of peppermint schnapps. "Oh," he replied, "a friend of mine recommended it as a great "Mexican drink." The questioner responded that it is a German product which is very popular in that country. The man with the bottle appeared puzzled, perhaps thinking that information received from his informant was usually quite reliable. To his interrogator he responded, after a moment's hesitation, "How ***sure*** are you?"

Individuals, being only human, often reveal deficiencies in plausible reasoning. On occasion, however, reasoning that at first appears to be faulty is not, as in many cases in advertising and public relations. On other occasions deficiencies are real. In such cases there may simply be a lack of appropriate skills. Chapter VIII is devoted to an analysis of common kinds of deficiencies in reasoning, both real and supposedly real. Here, however, the emphasis is upon the characteristics of sound reasoning.

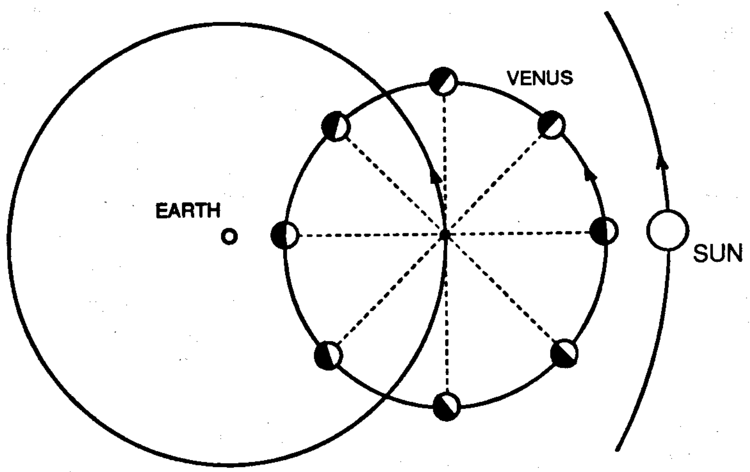
Introductory Problems

PHASES OF VENUS. In the sixteenth century the prevailing view as to the arrangement of the sun, moon, and planets was the Ptolemaic view that the earth was at the center of this system with the moon orbiting about the earth relatively close by and with the sun and planets in orbits about the earth farther out. In 1543, however, the Polish astronomer Copernicus proposed a new arrangement with the sun, not the earth, at the center of things. Copernicus thus hypothesized a heliocentric system to replace the geocentric model. Observations at the time neither contradicted nor supported one view of the planetary system or the other. Was the sun at the center of the universe or was it the earth? It wasn't until the year 1609 when the first telescope was invented that observational evidence could be brought to bear to settle the question of whether the Copernican system or the older Ptolemaic system was correct.

In the Ptolemaic universe the earth is at the center and the moon, planets, sun, and stars occupy a series of concentric rings about the earth. In the years prior to the time that Galileo first viewed the skies with a telescope, speculation was rampant regarding the two models, the earth system and the solar system. One of Galileo's students suggested to him that if the Copernican system was correct, then Venus, which lies between the sun and the earth, should show all the same phases as the moon, from a thin crescent through quarter full, to full, and back again to a waning crescent. In consequence, the student continued, if Venus actually does show phases like the Moon, the new sun-centered model must surely be correct. The problem here is whether to agree or disagree with the student that his stated conclusion is correct.

Before examining the student's conjecture one must understand that at the time the simple model with the moon, sun, planets, and stars in circular orbits about the earth had been abandoned in favor of a more sophisticated version of the Ptolemaic model in which the moon and planets did not themselves move in circles about the earth. Instead, they were presumed to be attached to giant wheels whose axles moved in circles about the earth. This modification was necessary to account for the so-called retrograde motion of the planet Mars and also, the fact that Venus never appeared in the sky at an angle greater than 48° from the direction to the sun. The motion of Venus in this model relative to the earth and to the sun is diagrammed below.

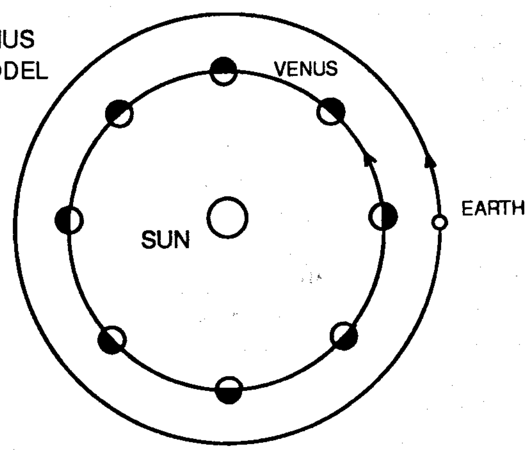
PHASES OF VENUS — PTOLEMAIC MODEL



According to this model, starting with Venus nearest the sun, an observer on earth would first see the dark side of Venus and later a series of crescent shapes followed by another view of the dark side followed by the reverse sequence of crescent shapes for Venus.

The student must have realized this. Recall that he said that *if* Copernicus was right in thinking that the sun, not the earth, was at the center of things, *then* Venus would show all the phases of the moon, crescent shapes leading up to half full and gibbous shapes and from there to full. A heliocentric model showing only the earth, the sun, and Venus, is given below. This model gives moon-like phases for Venus.

PHASES OF VENUS
COPERNICAN MODEL



Of course the advent of the telescope did bring the moon-like phases of Venus into view. Does this mean, as the student claimed, that the Copernican model was indeed correct?

SLAVERY AND THE CIVIL WAR. The Civil War in America was a long and bloody struggle between the North and the South. There were a number of important factors that led to the beginning of the war when southern forces fired on Fort Sumter. Perhaps the most important factor contributing to the onset of war was the issue of slavery about which emotions ran extremely high. Let us assume that this single factor *strongly suggested* that military conflict was inevitable. The supposition, then, is that the issue of slavery nearly implied civil war as a consequence. A strictly hypothetical question is this: If there had been no civil war, could one have then concluded that the issue of slavery was not really considered to be of great significance? To fix ideas, let S represent the issue of slavery and CW the onset of civil war. Set the prior probability that the issue of slavery S was significant be $P(S) = 0.50$ and let $P(CW|S) = 0.90$ and $P(CW|\bar{S}) = 0.30$. Now find the value of $P(S|CW)$ and $P(S|\overline{CW})$. Can it be said that no civil war, $(\overline{CW})$, would strongly suggest that slavery was not a dominant issue, i.e., $\bar{S}$?

ANALOGY. In discussing the situation in which two properties are said to be analogous, Polya presents for comparison the data in the two tables shown below. In the table at the left are listed perimeters of certain plane figures, each figure having the same area. (This area has been set at 1.00.) At the right below are listed the fundamental frequencies of vibration of membranes stretched to cover certain plane geometric shapes, again each shape having the same area. The fundamental vibrational frequency of a circular membrane such as found in a kettle drum occurs when the entire membrane vibrates up and down in unison. In higher frequency modes of vibration part of the circular membrane moves upward at the same time that other parts are vibrating downward. Similar remarks could be made about the other shapes. What is important is that the fundamental frequency listed is the lowest possible vibrational frequency for that shape.

Perimeters		Frequencies	
Circle	3.55	Circle	4.261
Square	4.00	Square	4.443
Quadrant	4.03	Quadrant	4.551
Rectangle 3:2	4.08	Sextant	4.616
Semicircle	4.10	Rectangle 3:2	4.624
Sextant	4.21	Equilateral triangle	4.774
Rectangle 2:1	4.24	Semicircle	4.803
Equilateral triangle	4.56	Rectangle 2:1	4.967
Rectangle 3:1	4.64	Isosceles right triangle	4.967
Isosceles right triangle	4.84	Rectangle 3:1	5.736

The entries in each listing increase from top to bottom and while the first three plane figures, the circle, square, and quadrant, are the same in each table, later entries are not in the same order but not far from it. We say that the two tables are *analogous* to each other or, closely correspond to each other. While the listings are the same in many respects, they also differ in certain respects. According to the table it would seem that of all plane figures the circle has the least perimeter (circumference) for a given area. It had long been a conjecture that no other plane figure had a smaller perimeter. Later, it was proven, mathematically, that it was true. What can be said now about the conjecture that the circular membrane, of all plane membranes, vibrates in its fundamental mode at the very lowest frequency?

FOUND GUILTY / ACTUALLY GUILTY. One-hundred individuals are brought to trial. Some of these are found guilty (FG) and some are found innocent (FI). Some are actually guilty (AG) and some are actually innocent (AI). Given below are reasonable expectations of society for the judicial system:

FI

$P(AG) = 0.40$
$P(FG|AG) = 0.90$
$P(FG|A\,I) = 0.05$

Find the values for both $P(AG|FG)$ and $P(AG|AI)$. Can it be said that the statements that an individual is AG and that he is also FG are analogous, i.e., closely correspond to one another?

O sea, ¿las líneas de relevancias se aproximan ambos a la diagonal ?

Background

LOGIC IN LANGUAGE. In ordinary language we frequently use expressions that denote logical relationships between statements, conjectures, or propositions. Most of us probably don't realize how often we use this kind of language. We can also find it in newspapers, magazines, on TV, in advertising, in books, and in ordinary conversation. Listed below are excerpts from Tapscott's *Elementary Applied Symbolic Logic* in a chapter he calls "A Logic-English Translation Guide."

Conjunctive Operators

and	but
although	however
. . . also . . .	whereas
both . . . and . . .	but even so
after all	for
nevertheless	besides
not only . . . but also . . .	in spite of the fact that
plus the fact that	
still (*except in the sense of* 'any more')	
even though (*but not* 'even if')	
inasmuch as (*but not* 'insofar as')	
while (*in the sense of* 'although')	
since (*in the sense of* 'whereas', *but not* 'after')	
as (*in the sense of* 'whereas', *not* 'at the same time as')	

Note: Of the 21 entries in this list, the following five also do double duty as temporal indicators: *still, while, since, as,* & *and* (*when used in the sense of* 'and then')

Disjunctive Operators

or	either . . . or . . .
or else	or, alternatively
otherwise	with the alternative that
unless	

Conditional Operators

Forms in which the antecedent (. . .) comes before the consequent (- - -)

if . . . then - - -	if . . . , - - -
given that . . . it follows that - - -	given that . . . , - - -
not . . . unless - - -	in case . . . , - - -
insofar as . . . , - - -	so long as . . . , - - -
. . . implies - - -	. . . leads to - - -
. . . only if - - -	whenever . . . , - - -
. . . is a sufficient condition for - - -	. . . means that - - -
to the extent that . . . , - - -	

Forms in which the consequent (- - -) comes before the antecedent (. . .)

- - - if . . .	- - - in case . . .
unless - - - , not . . .	- - - whenever . . .
- - - insofar as . . .	- - - so long as . . .
- - - follows from . . .	- - - is implied by . . .
- - - is a necessary condition for . . .	only if - - - , . . .
- - - provided that . . .	- - - to the extent that . . .

Biconditional Operators

if and only if	if but only if
is equivalent to	is a necessary and sufficient condition for
just in case	just if
just insofar as	just to the extent that

This table lists phrases, called *operators,* that connect two statements or propositions. For example: The child has a high fever *and* a rash on her face. The defendant is likely to be fined *or* serve several weeks in jail *or* both. *If* we get no more rain this month *then* we'll set an all-time record for low rainfall. Getting an advanced degree *is equivalent to* a union card when it comes to getting a teaching position. Since the emphasis here is upon logical relationships among propositions and not on the propositions themselves, it is useful to denote the two propositions involved as proposition A and proposition B. The preceding statements can now be abbreviated to read as follows: A *and* B, A *or* B *or* both, *If* A *then* B, and A *is equivalent to* B.

Truth Table Representation. We start with the simplest possible kind of logic, that in which propositions are either true or false, i.e., T or F. In the truth table below the two columns at the left give all the possible combinatins of T and F. These combinations represent possible conjunctive relationships between propositions A and B. In the first row we have both A and B true, in the second row A is true while B is false, in the third row A is false and B true, and in the last row both propositions are false. These combinations lie to the left of the heavier vertical line. To the right of this line are the truth values, T's or F's, of these conjunctive combinations for the particular logical relationship indicated at the head of the column. Thus each vertical column of T's and F's forms a kind of fingerprint for a logical relationship. All T's in a column represents the relationship A*B, called complete affirmation. In the next column a T then an F followed by two T's represents A→B, etc. Symbols are used rather than words to denote logical relationships. Thus A→B stands for "if A then B," or what is the same thing, "A implies B," i.e. the truth of A implies the truth of B. A←B is written for "A if B" or "A is implied by B." A↔B represents A implies B *and* B implies A. It can also be stated as "A if and only if B" or "A is equivalent to B."

TRUTH TABLE

A	B	A*B	A→B	A←B	A↔B	A
T	T	T	T	T	T	T
T	F	T	F	T	F	T
F	T	T	T	F	F	F
F	F	T	T	T	T	F

The relationship of implication A→B gives many people difficulty when it is read as "if A then B." That relationship is evident in the first two rows where we get a T when both A and B are true but an F when A is true but B is false. What *isn't* evident is that when A is false, B can then be either true or false. The "if . . . then" language simply doesn't specify the relationship when proposition A is false. The relationship A←B can be thought of as B implies A which must be the same as A→B with A and B interchanged. The relationship A↔B can be stated not only as "A is equivalent to B," but also as "the truth of A is a necessary and sufficient condition for the truth of B." As can be seen from the truth table the relationship is T when both A and B are true and again when both are false, but is F otherwise. Heading the last column we simply have A, i.e., "A is true," which has truth values identical to those in the extreme lefthand column. A good way to look at this relationship is that A is true *no matter what* — no matter whether B is true or false.

As examples of the above logical relationships, relationships which can be described by their columns of T's and F's, consider the following translations into ordinary English:

S*W In April some days are sunny and warm (S.W), some are cloudy and warm ($\bar{S}.W$), some are sunny and cool ($S.\bar{W}$), while still others are cloudy and cool ($\bar{S}.\bar{W}$).

S→G If Henry gets the top score (S) in his psychology class he is then sure to receive a top grade (G).

B←F Mary's flowers will have many large beautiful blooms (B) provided that she fertilizes them regularly.

J↔M Janet and Mary are undecided whether they will go to the gymnastics meet. They will either both go (J.M) or neither will go ($\bar{J}.\bar{M}$).

H One's height (H) is in no way influenced by one's intelligence (I).

2X2 Matrix Representation. At this point we make a significant break with the supposition that propositions can only be true or false. We now take a proposition to have a certain probability for being true and a corresponding probability for being false. For proposition A, for example, P(A) is the probability it is true and P($\bar{A}$) the probability that it is false. Recall from Chapter II that P(A) + P($\bar{A}$) = 1. In a table similar to the one above, for each logical relationship reading from the top down the elements are P(A.B), P(A.$\bar{B}$), P($\bar{A}$.B), and P($\bar{A}$.$\bar{B}$). One example (and only one) of each of the logical relationships with which we have been working is shown in the probability table below.

PROBABILITY TABLE EXAMPLES

Conjunction	A*B	A→B	A←B	A↔B	A
P(A.B)	0.48	0.20	0.33	0.40	0.30
P(A.$\bar{B}$)	0.22	0	0.33	0	0.70
P($\bar{A}$.B)	0.12	0.35	0	0	0
P($\bar{A}$.$\bar{B}$)	0.18	0.45	0.33	0.60	0

The probability values entered in the above table for each of the five relationships shown between propositions A and B are obtained as follows. First, 0's are placed in each box if the corresponding truth table location that has an F in it. Other than this, the non-zero entries in each column must add to one. Any set of values that satisfies these two conditions is an example of the logical relationship specified at the top of the column. An immediate conclusion is that there is not just one A*B relationship, nor just one A→B relationship, nor just one of any of the others, but many relationships all qualifying as instances of a general characteristic. That general characteristic is distinguished from the others by the number and pattern of 0's contained in the column of four values. Note that in the first column there are no 0's, the next two have a single 0, and the last two columns both have two 0's. From the probability table representation it is just a small step to obtain 2X2 matrices that describe the same relationships between propositions A and B.

2X2 PROBABILITY MATRIX EXAMPLES

	$\bar{B}$	B
A	0.22	0.48
$\bar{A}$	0.18	0.12

A*B

	$\bar{B}$	B
A	0	0.20
$\bar{A}$	0.45	0.35

A→B

	$\bar{B}$	B
A	0.33	0.33
$\bar{A}$	0.33	0

A←B

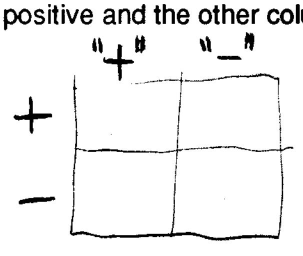
	$\bar{B}$	B
A	0	0.40
$\bar{A}$	0.60	0

A↔B

	$\bar{B}$	B
A	0.70	0.30
$\bar{A}$	0	0

A

The probability values in these matrices are the same as those in the columns in the Probability Table examples. The matrices have more graphic appeal than the column arrangement because each conjunctive probability lies at the intersection of the row and column denoting the conjunction. In addition, a matrix representation of the logical relationship between two propositions is natural since observational data is often taken in this form. For example, in the Blue and Green Taxicab problem in Chapter II one row could represent a Blue taxicab and the other row not a blue taxicab, i.e. a green cab. One column could represent the eyewitness identification of the cab as blue and the other column its identification as not blue. A 2X2 matrix representation is also appropriate in the Mammalary Cancer problem. Here, the two rows in the matrix could represent having cancer and not having cancer while one column represents a test result that is positive and the other column one that is negative.



Graphical Representation. As appealing as the 2X2 matrix representation is for some purposes, it does not focus upon a number of parameters that are often of great interest. For example, what is the probability of A, i.e., what is P(A)? What is P(B)? There are in addition four conditional probabilities that enter into many problem situations. These are P(A|B), $P(A|\bar{B})$, P(B|A), and $P(B|\bar{A})$. As you will recall, conditional probabilities play an essential role in Bayes' equation. Each of these six parameters, let us call them *Bayes' parameters*, can be determined without too much difficulty from the four conjunctive probabilities that are entered in the 2X2 probability matrices. This has been done in obtaining the values listed below for each of the same five numerical example relationships with which we have been working.

THE BAYES' PARAMETERS

	A*B	A→B	A←B	A↔B	A
P(A)	0.70	0.20	0.67	0.40	1.00
P(B)	0.60	0.55	0.33	0.40	0.30
P(A|B)	0.80	0.36	1	1	1
$P(A\|\bar{B})$	0.55	0	0.50	0	1
P(B|A)	0.69	1	0.50	1	0.30
$P(B\|\bar{A})$	0.40	0.44	0	0	—

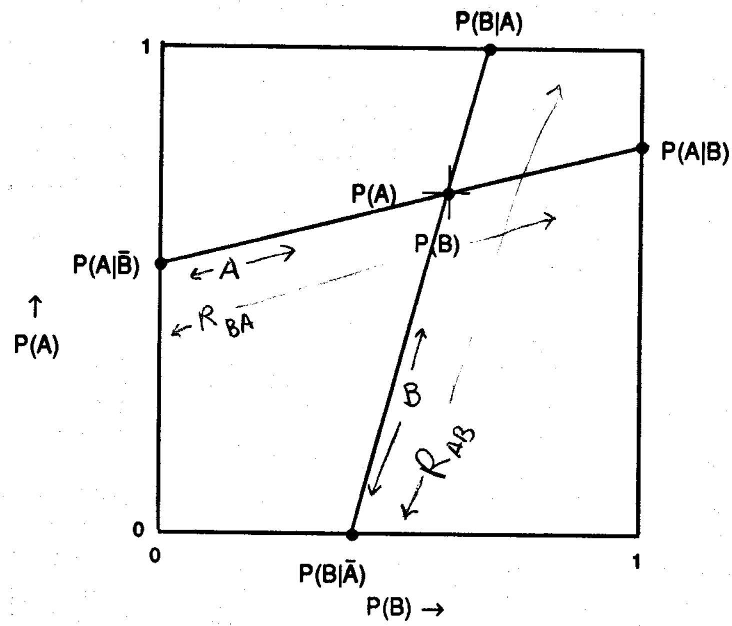
Let's check how the values for each of the six Bayes' parameters for the relationship A*B are obtained from the conjunctive probabilities in the 2X2 matrix. P(A) is simply the sum of the two probabilities in the top row of the matrix since $P(A) = P(A.B) + P(A.\bar{B})$. Likewise, P(B) is the sum of the two probabilities in the right hand column. To determine P(A|B) we focus on the right hand column since it is there that the condition B (B true) is satisfied. The upper of these two values represents proposition A being true, so P(A|B) is this value in the upper right hand corner of the matrix, P(A.B), divided P(B), the sum of the two values in the right hand column. Numerically, P(A|B) = 0.48 / (0.48 + 0.12) = 0.80. In similar fashion the remaining three conditional probabilities in the A*B column above are: $P(A|\bar{B})$ = the upper left box value divided by the sum of the values in the left hand column; P(B|A) = upper right box value divided by the sum of the values in the top row; and finally, $P(B|\bar{A})$ = lower right hand box value divided by the sum of the values in the bottom row.

medir

It remains to exhibit the six Bayes' parameters in graphical form. Since we are dealing with two propositions only, each of which can assume a probability value between zero and one, it seems quite natural to create a graph that extends from 0 to 1 along a horizontal axis and also from 0 to 1 along a vertical axis. Let us plot P(A) vertically and P(B) horizontally to locate a point whose coordinates are P(B) and P(A). This is exactly the same as plotting a point whose coordinates are x and y on a sheet of graph paper. Having done this, we now recognize that the quantities P(A|B) and $P(A|\bar{B})$ are simply two kinds of P(A)'s, one conditioned on the truth of B and the other on its falsity. Thus P(A|B) should represent a point plotted in the vertical direction at the location where B is true, i.e., where P(B) = 1, and $P(A|\bar{B})$ another point, also plotted vertically, but from the location where B is false, i.e., where P(B) = 0. These two points therefore lie on opposite side edges of our probability square. The quantities P(B|A) and $P(B|\bar{A})$ become quantities plotted to the right from the left side of the square, one along the top of the square and one along the bottom side. This procedure locates five points on our graph as illustrated below. Let us call such a graph a *Bayes' Diagram*.

BAYES' DIAGRAM

GRAPHICAL
REPRESENTATION
OF
A*B



Note that when the points $P(A|B)$ and $P(A|\bar{B})$ are connected by a straight line and the points $P(B|A)$ and $P(B|\bar{A})$ are connected by a second line, these two lines intersect at the point whose coordinates are $P(B)$ and $P(A)$. For reasons that will be made clear later, these lines will be referred to as *relevance lines*. The locations of the five points shown above are to scale where $P(A) = 0.70$, $P(B) = 0.60$, $P(A|B) = 0.80$, $P(A|\bar{B}) = 0.55$, $P(B|A) = 0.69$, and $P(B|\bar{A}) = 0.40$. This is the same A*B relationship that was previously represented in a probability table and again in a probability matrix. The graphical representation of the logical relationship between propositions A and B has two important features. First, the nature of a logical relationship is exhibited by the geometry of the diagram providing a new perspective to the relationship. Second, it not only represents all six Bayes' parameters in a geometrical way, but permits the analysis of the **dynamics** of a changing relationship between two propositions, conjectures, or statements.

Shown below are Bayes' diagrams for the first four logical relationships with which we have been dealing (all to the same scale) together with four additional relationships. By displaying all eight, each can readily be compared to the others. When a relevance line terminates at a corner of the square that point is marked by a large dot and labelled. In such cases the arrangement of these points serves to identify the particular logical relationship that is being represented.

Diagrams 1, 2, 3, and 4 are graphical representations of the logical relationships A*B, A → B, A ← B, and A ↔ B. A*B displays the fact that none of the conjunctive probabilities is zero in this case because neither of the relevance lines connects to any of the corners of the square. For A → B, however, each of the lines terminates at one corner of the square.

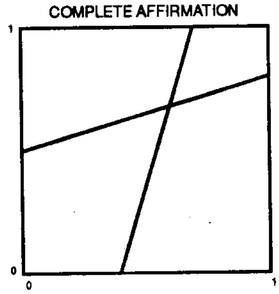
Diagram 3 shows converse implication, i.e., A is implied by B. Notice the difference between this and diagram 2. In both cases the dots anchor one end of each relevance line. In diagram 4 the two relevance lines are superimposed and become one. This diagram displays equivalence, or, A implies B **and** B implies A.

Diagram 5 shows that proposition A is independent of proposition B and consequently B also is independent of A. This diagram is related to diagram 1. The only difference is that here the relevance lines are horizontal and vertical. Diagram 6 is clearly related to diagram 4 directly above it. The single bar over the symbol shows that the entire relationship A ↔ B is negated, which, in a truth table representation would turn every T into an F and every F into a T.

Diagrams 7 and 8 are clearly related to diagrams 2 and 3. Truth table representations for the four relationships would contain three T's and one F. One conclusion from this is that every one of the four can be seen as an *or* relationship and every one of the four can also be seen as a kind of implication. The four are distinguished by that corner of the square that seems to be avoided by the two relevance lines.

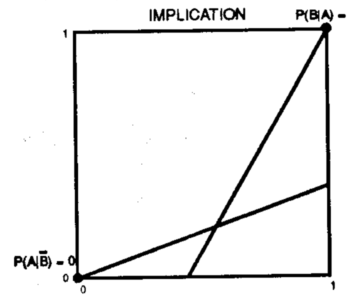
1. A*B

COMPLETE AFFIRMATION



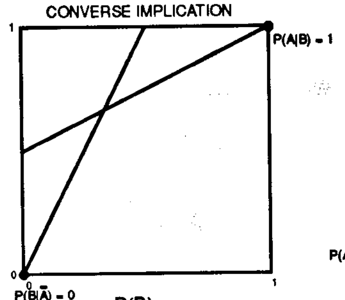
2. A → B

IMPLICATION



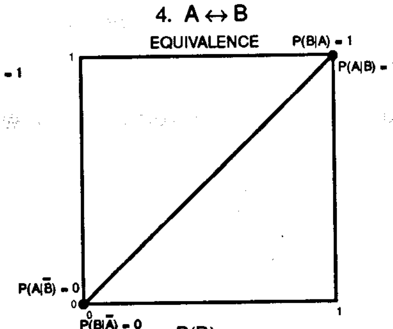
3. A ← B

CONVERSE IMPLICATION



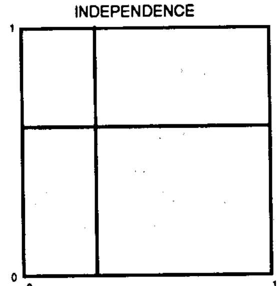
4. A ↔ B

EQUIVALENCE



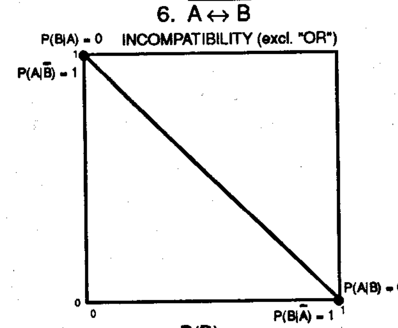
5. $R_{BA} = R_{AB} = 0$

INDEPENDENCE



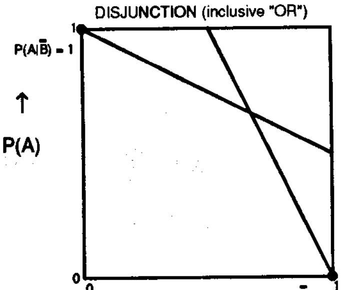
6. $\overline{A \leftrightarrow B}$

INCOMPATIBILITY (excl. "OR")



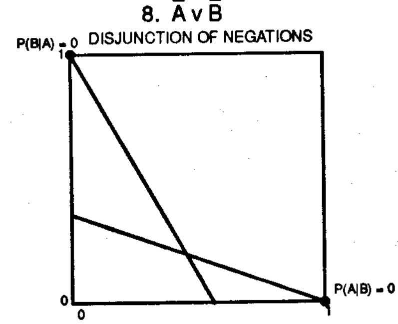
7. A v B

DISJUNCTION (inclusive "OR")



8. Ā v B̄

DISJUNCTION OF NEGATIONS



In summary, we now have three different kinds of probabilistic schemes to represent a logical relationship between two propositions A and B. Let us compare the three using the same logical relationship A*B that has been described previously. The three are shown below.

2x2 Matrix

	$\bar{B}$	B
A	0.22	0.48
$\bar{A}$	0.18	0.12

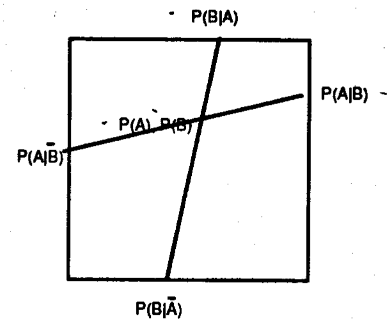
A*B

Bayes' Parameters

P(A) = 0.70
P(B) = 0.60
P(A|B) = 0.80
P(A|$\bar{B}$) = 0.55
P(B|A) = 0.69
P(B|$\bar{A}$) = 0.40

A*B

Bayes' Diagram



A*B

For those who are digitally inclined, either the 2x2 matrix representation or that provided by a listing of Bayes' parameters may be preferable to that of the Bayes' diagram. Of these, the 2x2 matrix accommodates data in a natural way while the Bayes' parameters may be superior in many instances of problem solving involving Bayes' equation. The geometrical representation provided by the Bayes' diagram will most likely be preferred by those who are more geometrically oriented, which includes most of us.

PLAUSIBLE REASONING

Previously displayed is Tapscott's "A Logic-English Translation Guide." It refers to logical relationships that can be described by a column of truth-table entries. Moreover, these translations into ordinary English apply to a two-value system of logic in which propositions are either true T or false F. Seldom in ordinary experience does one encounter relationships that can be described in this way. While individuals often speak in a language which seems to consist of statements that are either T or F, when pressed they will readily agree to certain qualifications. For example, "always" becomes "almost always," "never" becomes "seldom," and " true" becomes "almost always true." Qualifications of one kind or another transform deductive logic into plausible reasoning.

A partial list of words and phrases that are used to soften the absolutes of deductive logic include:

usually	rarely	relevant	is proportional to	analogous	confirms
nearly	irrelevant	probably	likely	possibly	credible
almost	approximately	affirm	unlikely	plausible	is related to
suggests	reasonably	equivalent	persuasive	similar	tends to

Qualiifiers convert the language of deductive logic into that of plausible reasoning:

S*W qualified: Some days now are mostly sunny and fairly warm, some are mostly cloudy and fairly warm, some are mostly sunny and rather cool, while still others are rather cloudy and cool.

S→G qualified: If Henry gets the top score in his psychology class he is almost guaranteed an A.

B←F qualified: Mary's flowers will probably have many large beautiful blooms provided that she fertilizes them but doesn't overdo it.

J↔M qualified: Janet and Mary will probably both go to the gymnastics meet or neither will.

H qualified: One's intelligence (I) in all liklihood has no influence on one's height (H).

Learning. Recall that Bayes' equation can be interpreted as a *learning equation.* Writing Bayes' equation as shown below,

posterior ↘ ↙ prior

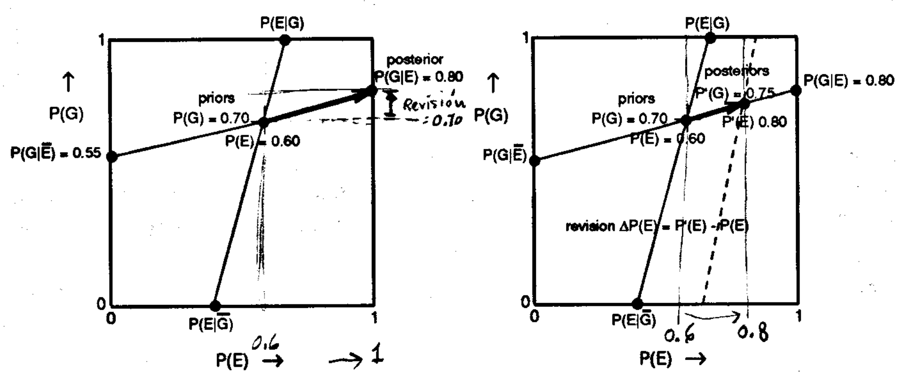
$$P(A|B) = \frac{P(B|A)}{P(B)} P(A)$$

P(A) is interpreted as the prior probability of proposition A, meaning that it is the appropriate value before additional information has been gained about P(B). P(A|B), on the other hand, is taken to be a revised value of P(A) having now *learned* that B is true. It is therefore called the posterior probability value for A. This same relationship can be seen using a Bayes' diagram of the relationship between A and B as shown at the left below. Take, for example, the relationship A*B with numerical values as previously given. This time, however, let us take proposition A to be proposition G, the probability that a certain defendant is guilty of the charges against him. Proposition B is now additional evidence E which tends to further incriminate the defendant. The posterior probability of guilt of the defendant is given by the learning equation below and by the Bayes' diagram shown at the left the left below.

$$P(G|E) = \frac{P(E|G)}{P(E)} P(G)$$

Revision. With this perspective we see from the diagram at the left that, having learned that the evidence E is true, Bayes' equation moves the point at which P(G) = 0.70 to the point P(G|E) = 0.80. This revision is indicated by the heavy arrow on the diagram. Having confirmed the evidence E to be true it can be concluded that the defendant is now more likely to be guilty than he was before. His probability of guilt has increased from 0.70 to 0.80. Of course a downward revision in P(E) would lead to a decrease in the probability that the defendant is guilty.

Revision to P(E) = 1   Revision to P'(E)



What if the evidence is not confirmed? What if the evidence E has become more credible, but not so credible as to be called "true"? Suppose P(E) increases, not from 0.6 to 1.0, but from 0.6 to 0.8? This is just half the revision in the probability of the evidence as when it was found to be completely true. This situation is illustrated in the figure at the right above where the heavy arrow indicates the result of a smaller revision in the probability for the evidence to be true. P(G) still increases as a result of this revision, but by less than when evidence E became certain. As shown to scale in the diagram at the right, the posterior probabilities are P'(E) = 0.80 and P'(G) = 0.75. The revision in the probability of the evidence has as a consequence the increase in the probability that the defendant is guilty. Other evidence could, of course, lead to a downward revision in the probability of guilt. It is by patterns such as this that we continually revise our opinions about all sorts of things based upon what we learn. We have now moved out of the realm of deductive logic into that of plausible reasoning.

Relevance. The process of revision can be better understood by introducing a quantity called *relevance*, a term appropriate to the field of plausible reasoning as well as to trial law. Referring to the graphical representation of G*E above, it is seen that the increase in P(G) as proposition B becomes more credible depends on two things. It depends on the slope of the line that connects the points $P(G|E)$ and $P(G|\bar{E})$ and upon the amount of increase in P(E). The slope of this line can be defined by the quantity $P(G|E) - P(G|\bar{E})$, i.e., by the "rise" in the line across the square from the left side to the right side, divided by the "run." But the "run" is one,since that is the width of the square. So we are left with the definition for the relevance of the evidence E to the question of guilt G as follows:

$$R_{EG} = P(G|E) - P(G|\bar{E})$$

It can now be understood why the line connecting the points $P(G|E)$ and $P(G|\bar{E})$ is called a *relevance line*. In the revisions in the evidence as described on the Bayes' diagrams above the revised value of P(G), whether it be P(G|E) when P(E) has become one, or P'(G) when the revision is smaller, these new values lie on the more horizontal relevance line as did the prior values P(G), because the relevance $R_{G|E}$ is constant. This relevance is a property of the relationship between the evidence and the proposition of guilt. Some evidence is more relevant to Guilt than others. Relevance values range between -1 and 1. The relevance of E to G in the situations described by the Bayes' diagrams above is +0.25.

We could, of course, talk about the relevance of Guilt G to the evidence E. It is defined in an analogous fashion to that of evidence E to guilt G. Its value is given by:

$$R_{GE} = P(E|G) - P(E|\bar{G})$$

It is the slope of the more vertical relevance line in the previously given Bayes' diagrams, but only if one measures that slope relative to a vertical, not a horizontal, line. In assessing the impact of some piece of evidence E to the probability of guilt the relevance R_{GE} doesn't enter in. Nonetheless, it is clear that guilt or innocence is relevant to the truth or falsity of the evidence. A certain piece of incriminating evidence, for example, may make much more sense given that a suspect is guilty than if he's not.

Later, in the problem of resolving the PARADOX IN DIAGNOSTICS, the probability for having a certain disease D is plotted vertically on a Bayes' diagram and the probability for a positive result for a certain diagnostic test is plotted horizontally. A patient is successively given a number of independent tests for the disease all of which have the same relevance for D to a positive outcome for the test. In this case, in contrast to the situation in which evidence is relevant to guilt or innocence, it is the slope of the more vertical relevance line that remains constant. The conclusion from this is that a problem solver should assess each situation carefully to determine which relevance, if any, stays constant when revising a probability according to Bayes' equation. When such a relevance is identified Bayes' equation can be applied successively to follow the dynamics of a logical relationship.

LOGICAL RELATIONSHIPS IN PLAUSIBLE REASONING. Plausible reasoning involves the analysis of logical relationships that are not quite A → B, only approximately A ← B, nearly A ↔ B, or almost A (being true). These are qualitatively different from the versions of the logical relationships first presented in truth table form. These are the relationships of plausible reasoning, the kind of reasoning that characterizes ordinary discourse. Examples of strict implication, total equivalence, or certain truth of any proposition are seldom encountered. Instances in which people make sound arguments, draw plausible conclusions, and establish reasonable relationships between conjectures are found everywhere. How are these processes to be described? Certainly not by truth tables, nor by 2X2 matrix representations. Of the different representation schemes presented, only one is well adapted for the description of relationships in plausible reasoning. It is the graphical representation scheme. Except for relationships 1 and 5 in the prior display of eight logical relationships, two of the ends of the relevance lines terminate at a corner of the Bayes' diagram. At these corners a conditional probability value is either 0 or 1. There are extremely important relationships in plausible reasoning in which the ends of one or both relevance lines come close to a corner of a Bayes' diagram but do not terminate there. In such cases the conditional probabilities at these corners come close to either zero or one. It is reasonable to describe these relationships as being close to implication, close to converse implication, close to equivalence, close to complete incompatibility, close to disjunction, or close to the disjunction of negations, all of which are described three pages earlier.

On the page following, under the title YOU ARE THE PROSECUTING ATTORNEY, four relationships in plausible reasoning are illustrated by way of their Bayes' diagrams. There is no "truth" here, only relevance and credibility. Each of the four diagrams is characterized by the orientation of the pair of relevance lines it exhibits. The values of the end points of these lines and the coordinates of their intersections satisfy Bayes' equation. Since there are an infinite number of sets of six Bayes' parameters, there are likewise an infinite number of logical relationships in plausible reasoning.

On the second page following there are instructions for CONSTRUCTING A BAYES' DIAGRAM. Bayes' equation was introduced in Chapter II and used there to analyze the Blue and Green Taxicabs problem, the problem of the patient with Mammalary Cancer, the archeological problem Blasting a Hypothesis, the problem called The Diagnostic Value of Acne, the X-Linked Lethals problem, and testing for the presence of the HIV virus. A Bayes' diagram could be drawn for each of these problems to provide a graphical perspective in addition to an algebraic one.

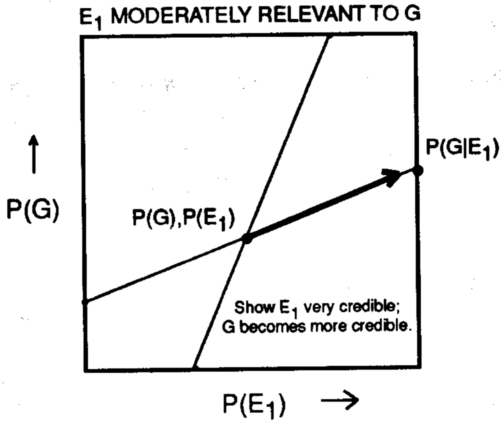
YOU ARE THE PROSECUTING ATTORNEY

A defendant in a robbery case has been bound over for trial primarily because an eyewitness saw him leave the scene shortly after the robbery occurred. It is also possible that the defendant is the cat burglar who presumably accounts for a number of recent unsolved robbery cases. You take the prior probability of his guilt to be $P(G) = 0.4$ and the prior probability that he is the cat burglar to be $P(C) = 0.4$. Investigators have now identified three incriminating pieces of evidence E_1, E_2, and E_3.

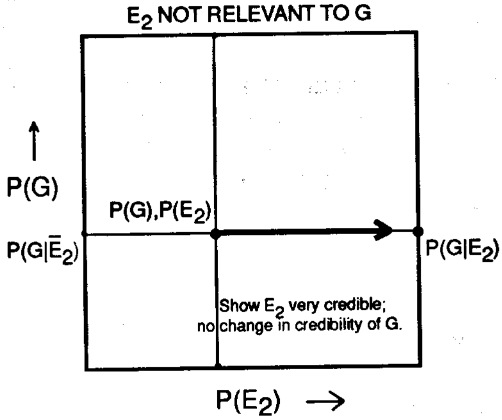
E_1: A partial fingerprint found at the scene is consistent with the defendant's fingerprint.
E_2: The defendant is a dark complexioned Middle Easterner.
E_3: A cat burglar costume found in the defendant's trash barrel belongs to him.

As the prosecuting attorney you now sketch four Bayes' diagrams starting in each case with two prior probability values (locating the intersection of the two relevance lines) and one conditional probability. During the trial it is your job as prosecutor to convince the jury that each of the three pieces of evidence is highly credible, i.e., warrants a probability increase indicated by a heavy arrow on the diagram. You will also want to persuade the jury that the probability of the defendant being the cat burglar is very high because this will also point strongly toward the defendant's guilt. If the defendant really is the cat burglar, that would suggest, i.e., nearly imply, the credibility of evidence E_3. However, as the last of the four diagrams shown below indicates, just because the costume is his by no means assures you that he is the cat burglar, for such a costume is a popular one at masked balls. Finally, you hope as prosecutor that you can convince the jury that when the evidence is compounded they will see fit to bring in a guilty verdict.

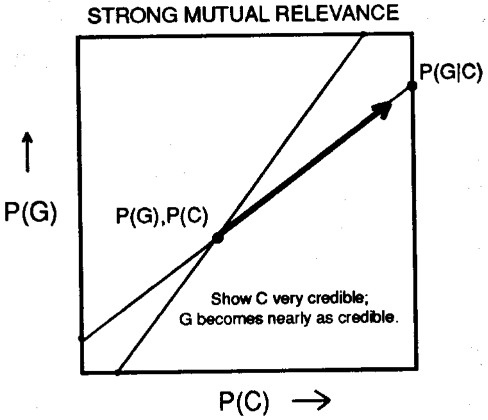
E_1 MODERATELY RELEVANT TO G



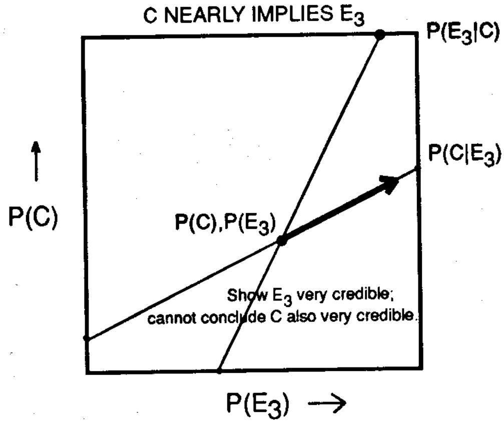
E_2 NOT RELEVANT TO G



STRONG MUTUAL RELEVANCE



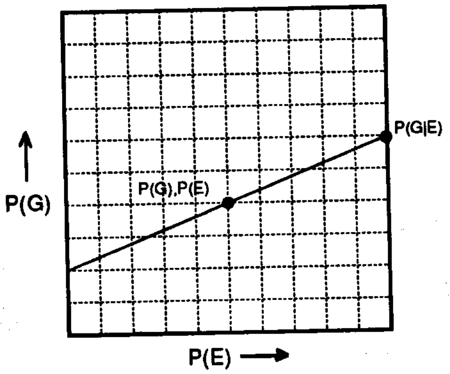
C NEARLY IMPLIES E_3



CONSTRUCTING A BAYES' DIAGRAM

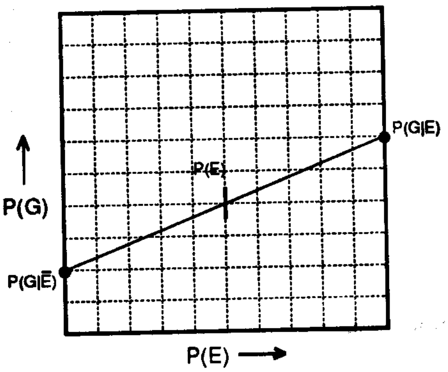
Let the proposition that a certain defendant is guilty be G and a certain piece of evidence that implicates him in a crime be E. Six Bayes' parameters describe the relationship between guilt G and the evidence E. To construct a Bayes' diagram one must specify three of these six parameters before the remaining three can be determined. Shown below at the left are three ways to initiate a graphical procedure for making this determination.

§1.1



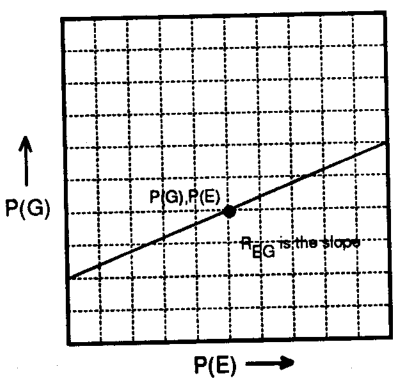
Here the three specified quantities are P(G) = 0.4, P(E) = 0.5, and P(G|E) = 0.6, each point being indicated by a heavy dot. A line connecting these two points is extended to the left side of the square where it defines the point P(G|Ē). Note that this procedure eliminates the need for finding P(G|Ē) algebraically.

§1.2



Alternatively, the two conditionals P(G|E) = 0.6 and P(G|Ē) = 0.2 can be specified together with one of the priors, in this case P(E) = 0.5. On the line drawn between these two conditionals the value of P(E) is marked by a short vertical line segment. Note that the intersection of the relevance line with the short vertical line segment graphically determines the value of P(G) to be 0.4, thus eliminating the necessity for finding P(G) algebraically.

§1.3



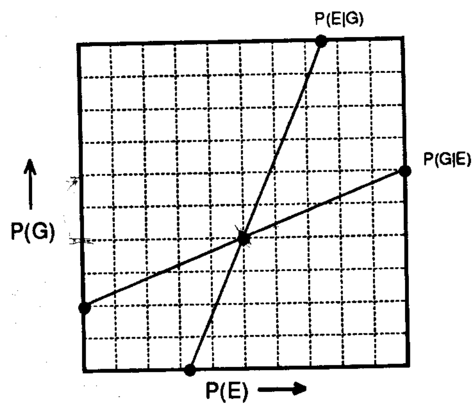
A third set of specified parameters consists of the two priors P(G) = 0.4 and P(E) = 0.5 together with the relevance of proposition E to proposition G, R_{EG} = 0.4. This relevance is the slope of the line drawn through the point P(G),P(E). Once again, the construction of the relevance line through the point representing the priors replaces algebraic methods for finding both P(G|E) and P(G|Ē).

§2.

Finally, as a second step following any one of the three beginning steps above, the value of P(E|G) is obtained using Bayes' equation:

$$P(E|G) = \frac{P(G|E)P(E)}{P(G)} = 0.75$$

The line drawn from P(E|G) through the point representing the priors and extending to the bottom of the square defines P(E|Ḡ). All six Bayes' parameters are now known.

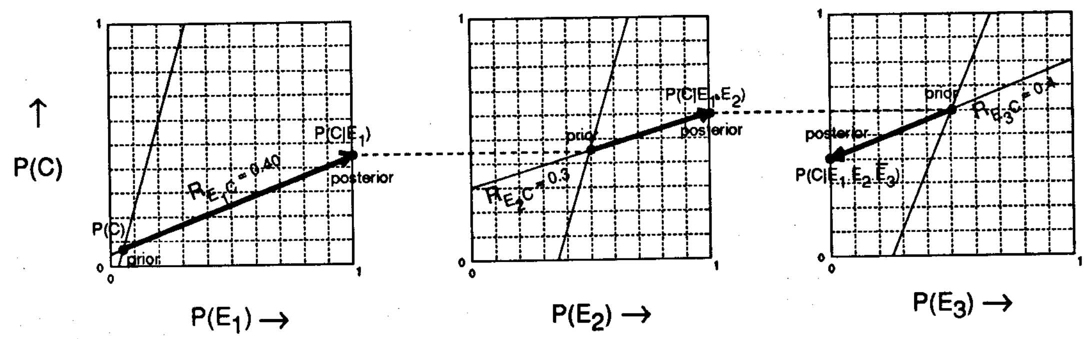


SUCCESSIVE VERIFICATIONS / MULTIPLE EVIDENCE. Researchers test a hypothesis through the examination of as many consequences of this hypothesis as can be identified. As each ~~hypothesis~~ is confirmed the probability that the hypothesis is true does nothing but increase. However, should any one consequent be found to be false the hypothesis is blasted, that is, is necessarily false. A second situation involves criminal cases in which the prosecuting attorney collects as many pieces of evidence as he can that implicate the defendant in the crime. So long as successive pieces of evidence point toward the guilt of the defendant, the probability that the defendant is guilty increases. The defense attorney, on the other hand, points out to the jury that certain other pieces of evidence tend to show that the defendant is innocent. The jury has the problem of assessing all the pieces of evidence. Proceedings in a criminal trial is a very special case of learning in general. In the learning process we continually revise our opinions on the basis of new information. In a third setting a person may be subject to a number of medical tests to determine whether he does or does not have a certain disease. The greater the number of tests that turn out to be positive, the more likely it is that the person has the affliction. Multiple evidence and successive verification of consequences play an extremely important role in plausible reasoning. We are not dealing here with the proposition that a defendant is 100% innocent or 100% percent guilty. Nor is it ever said that a large number of verified consequences actually *prove* a hypothesis. What is asserted in such cases is the high probability of that hypothesis.

The Copernican Model. As an example of these kinds of processes consider the evidence that tended to show that the geocentric model of the planetary system was incorrect and should be replaced with a heliocentric model as proposed by Copernicus. There was a wealth of evidence that supported the Copernican model and also a sizeable number of arguments that defended the Ptolemaic model. To simplify matters, in the analysis that follows only three pieces of evidence are considered. Two of these support the Copernican model and one does not. Let the propositions be:

- C: The Copernican sun-centered model of the planetary system is correct.
- E_1: Venus shows phases like the Moon.
- E_2: At times Mars is observed to move relative to the backdrop of stars in a backward direction.
- E_3: For the Copernican model the nearest stars, depending on the season, should appear in slightly different directions relative to more distant stars. This is called *parallax.*

Bayesian analysis involves the sequence of diagrams shown below, one for each of the relationships between proposition C and the pieces of evidence E_1, E_2, and E_3.



Analysis begins with the prior P(C) at the lower left corner of the first diagram. This value is low because the Copernican model went against the beliefs of the philosophers of the time and those of the Roman Catholic church. The first posterior $P(C|E_1)$ is obtained using the relevance value R_{E1C}. (This revision and those to follow are shown by bolder line segments.) This posterior becomes the prior in the second diagram. Using the relevance R_{E2C} one obtains the posterior $P(C|E_1.E_2)$ which in turn becomes the prior in the third diagram. Where before E_1 and E_2 were confirmed, now E_3 is found to be false—no parallax was observed. This accounts for the third posterior $P(C|E_1.E_2.\bar{E}_3)$ having a lower value than its prior. Although no parallax was observed in Copernicus' time, several centuries later its very small value was in fact observed, also showing that the closest stars lie well beyond the limits of the solar system.

¿Por qué no llega hasta 1? ... $P(C|E_1) \neq 1$, sólo si hay alguna otra hipótesis auxiliar(es) que 'salve(n) las apariencias', i.e. que explique(n) las fases de Venus sin renunciar a la tesis geocéntrica

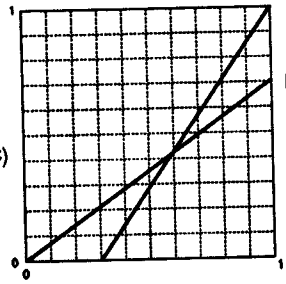
Introductory Problem Solutions

most likely

PHASES OF VENUS. RECAP PROB. The student's speculation was as follows: "If the Copernican system is correct, then Venus will [most likely] show phases, and if Venus does actually show phases, then it can be concluded that the Copernican system is correct."

DIAGRAM RELATIONSHIP. The first part of this statement contains the "if . . . then" language of implication. Let statement C be the hypothesis that the Copernican system is correct and statement Ph be the statement that Venus shows phases like the Moon. Thus the student's speculation can be written $C \rightarrow Ph$. This relationship can be represented on a Bayes' diagram as shown below. Note that the relationship between Ph and C that is shown is an example of deductive logic, not of plausible reasoning.

C→Ph



CONCLUSION. Since the value of P(C|Ph) shown on the right side of the diagram need not be particularly close to one, we can by no means conclude that the Copernican system is correct. That is, if Venus does show phases like the Moon, we cannot conclude from this that the Copernican system is necessarily correct. This, of course, conflicts with the student's conclusion.

SLAVERY AND THE CIVIL WAR. SET UP PROB. If we let CW represent the occurrence of the civil war, and S stand for the significance of the issue of slavery, then the assumption is that slavery strongly suggested that civil war would follow. To represent this situation let $P(S) = 0.50$, $P(CW|S) = 0.90$, and $P(CW|\bar{S}) = 0.30$. (Esta solución no sigue las instrucciones sugeridas en la p. 147, § 1.2, las cuales evitan la solución algebraica de P(CW))

SOLVE ALGEBRAICALLY. First find $P(CW) = P(CW|S)P(S) + P(CW|\bar{S})P(\bar{S})$

NUMERICALLY: $P(CW) = 0.60$

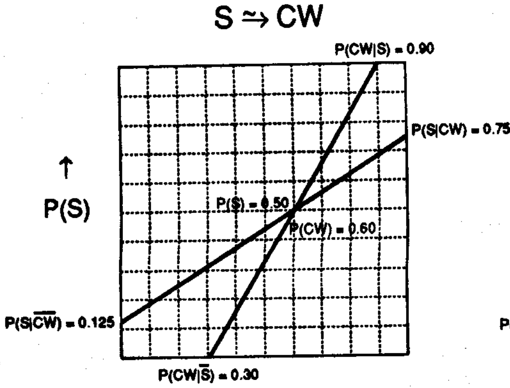
Second, find both $P(S|CW)$ and $P(S|\overline{CW})$:

$$P(S|CW) = P(CW|S)P(S) / P(CW) = (0.9)(0.5) / 0.6 = 0.75$$

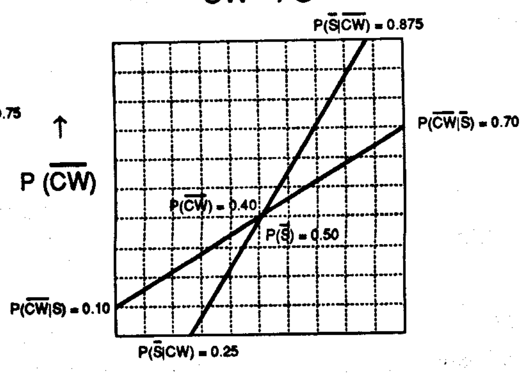
$$P(S|\overline{CW}) = P(\overline{CW}|S)P(S) / P(\overline{CW}) = (0.1)(0.5) / 0.4 = 0.125$$

SKETCH DIAGRAMS: (the above results are shown at the left below)

S ⇝ CW



C̄W̄ ⇝ S̄



La hipótesis que salva las apariencias, o mejor, salva la teo. ptolomaica de las apariencias, es la de que Venus (y Mercurio) se trasladan alrededor del Sol, en el trayecto de éste alrededor de la Tierra (Sistema Egipciaco, tychonico y Semi-Tychonico)

...s precisamente lo que ocurre con el contraejemplo del no-paralaje (E_3). ...ste fenómeno no descalabra definitivamente la teoría copernicana es porque ...ue puede explicarlo (salvarlo) a costa de aumentar brutalmente la distancia ... las Estrellas.

INTERPRET DIAGRAMS. Compare the orientation of the two relevance lines in the Bayes' diagram at the left above with the two that indicate that the Copernican model implies the statement that Venus will show phases like the earth's moon, i.e., that $C \rightarrow Ph$. The difference here is that in expressing the relationship between slavery and the civil war these relevance lines do not terminate at a vertex of the Bayes' square. One lies at a probability value only 0.10 from a vertex and the other 0.125 from a vertex. What can be said is that the S-CW relationship approximates the C-Ph relationship. In other words, the issue of slavery **strongly suggested** it would lead to war. Alternatively, one can say that the relationship between slavery and the civil war was **nearly** that of implication. That is why the relationship as noted above the diagram is written $S \rightsquigarrow CW$. The arrow of implication is accompanied by a squiggle to indicate the approximate nature of the relationship.

The Bayes' diagram on the right above plots $P(\overline{CW})$ vertically against $P(\overline{S})$ horizontally. All six Bayes' parameters for this relationship are simply taken from those describing the diagram to its left. When this is done we see a pair of relevance lines, though not identically situated to those on the left, are nonetheless indicative of a relationship that is nearly one of implication, but this time it is the relationship between the absence of civil war and the absence of slavery as a significant factor in leading to war. This relationship is written, as noted above its diagram, as $\overline{CW} \rightsquigarrow \overline{S}$. The answer to the question posed in this problem is now apparent. If there had been no civil war we could then conclude that the issue of slavery was not considered to be of great significance.

ASSESS PROBLEM SOLUTION. The two shorthand representations for these two diagrams obviously bear a relationship to one another. The pattern is as follows: first, A nearly implies B, and then the falsity of B nearly implies the falsity of A. A safe prediction from this result is that when $A \rightarrow B$, then it is also true that $\overline{B} \rightarrow \overline{A}$.

ANALOGY. RECAP PROB. It has now been proven that of all plane figures the circle has the least perimeter (circumference) for a given area. Because the listing of the lowest vibrational frequencies for many plane figures is so similar to a corresponding listing of lowest perimeters for plane figures, one would like to say that in all likelihood the lowest vibrational frequency is for a circular membrane just as the lowest perimeter is for a circular membrane.

FALSE CONCLUSION. Simply because the two tables are analogous to one another does *not* mean that we can conclude with *certainty* that the circular membrane vibrates with the lowest fundamental frequency. Two things that are analogous to one another means that they are similar, that is, the same with respect to many qualities but also different in some respects. The circular membrane just might be one of the shapes that is the exception to the overall similarity. It has already been noted that the two listings are not entirely in the same order from low to higher perimeter values and from low to higher frequency values. This is sufficient to make one wary about drawing an unjustified conclusion.

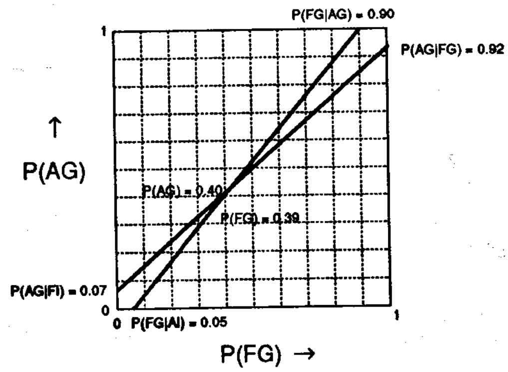
JUSTIFIABLE CONCLUSION. What can be concluded is that there is a strong possibility that the very lowest vibrational frequency is for a circular membrane. One's confidence in this conclusion strengthened when it was proved that the circle had the least perimeter for a given area.

FOUND GUILTY / ACTUALLY GUILTY. RESTATE PROB. One hundred individuals are brought to trial. Some of these are found guilty (FG) and some are found innocent (FI). Some are actually guilty (AG) and some are actually innocent (AI).

IDENT GIVENS AND WANTEDS. The expectations of society for the judicial system are given by the quantities at the left below. To the right are the remaining three Bayes' parameters as found using Bayes' equation.

$P(AG) = 0.40$	$P(FG) = 0.39$
$P(FG\|AG) = 0.90$	$P(AG\|FG) = 0.923$
$P(FG\|AI) = 0.05$	$P(AG\|FI) = 0.07$
	0.066

GRAPH RELATIONSHIP.

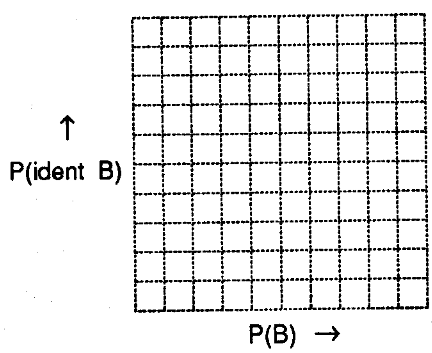


INTERPRET RELATIONSHIP. The above graph is very similar to one considered previously by the prosecuting attorney and labelled "strong mutual relevance." Also, referring to the collection of eight logical relationships given on page 141, we see that the Bayes' diagram above also closely resembles the one expressing equivalence between two propositions. In that case both relevance lines terminate in the lower left vertex of the square and in the upper right vertex. In the above, however, the relevance lines terminate fairly closely to the same vertices. We conclude that the relationship between AG and FG is nearly one of equivalence. This means that when an individual is certain to be guilty he has a high probability to be found guilty and when he is certain to be innocent he has a high probability to be found innocent. In an ideal, that is, perfect society, we would expect all those who are actually guilty to be found guilty and all those who are actually innocent to be found innocent. This would express the relationship of strict equivalence between AG and FG.

Additional Problems

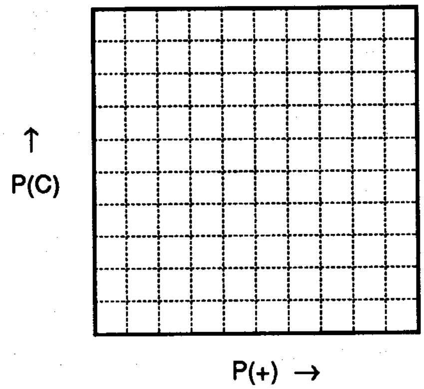
BLUE AND GREEN TAXICABS REVISITED. Recall that in the Blue and Green Taxicabs problem there was a hit and run incident involving a cab. In this rather small town 85% of the cabs are blue and the remaining 15% are green. An eyewitness identified the hit and run cab as blue. This same eyewitness was given a test under similar lighting conditions to determine the probability for her to be right in this identification. When the police used a blue cab for her to identify she correctly identified the color of the cab as blue 60% of the time. When a green cab was driven by for her to identify she said it was blue 20% of the time. Thus the data available upon the conclusion of these tests was as follows:

P(B) = prior probability that the cab was blue = 0.85
then P(G) = probability that the cab was green = 0.15
also P(ident B|B) = probability for the cab to be identified as blue on condition that it actually was blue = 0.60
and P(ident B|G) = probability for the cab to be identified as blue on condition that it actually was green (not blue) = 0.20

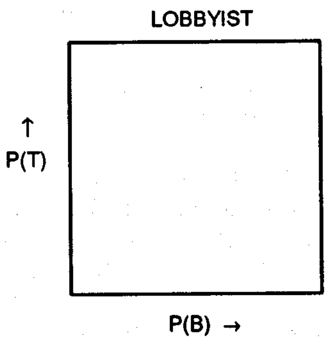


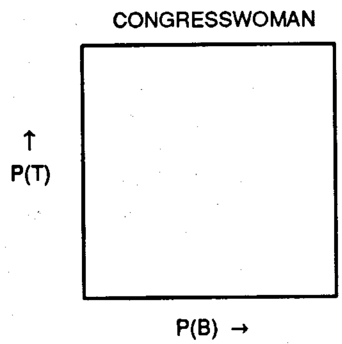
Construct a Bayes' diagram to represent the relationship between P(ident B) and P(B). Label all the Bayes' parameters. What is wanted in this problem, as before, is the quantity P(B|ident B), i.e., the probability that the cab really was blue on condition that it was identified as blue. What strategy should the defense attorney representing the Blue Cab Company adopt in the event this case goes to trial? What advantages and disadvantages does the graphical analysis of this problem have compared to the purely algebraic approach used earlier?

MAMMALARY CANCER PROBLEM REVISITED. The information that was supplied in this Chapter II problem consists of $P(C) = 0.10$, $P(+|C) = 0.79$, and $P(+|\bar{C}) = 0.10$. This, of course, is fictitious data. Really good data of the kind desired is simply not available.
The objective was to determine the inverse probability $P(C|+)$ which, having used Bayes' equation, turned out to be 0.48. For many individuals this set of numbers may not mean very much, particularly for those who are more graphically oriented. Repeat the analysis there, but this time find the remaining Bayes' parameters using graphical techniques where appropriate and plot these on a Bayes' diagram. The problem here is to analyse the accuracy of the tests for the presence of mammalary cancer. What is the rate for false positives? What is the rate for false negatives? Clearly, to improve the accuracy of the testing procedures both of these rates would have to be reduced. How would such reductions, if they could be accomplished, be reflected in changes in the Bayes' diagram?



***THE OIL LOBBYIST.** A lobbyist hired by the oil industry has taken Congresswoman Jones out to dinner, paid for her vacation to Bermuda, and arranged for her to obtain a good-sized loan from a bank at a favorable rate of interest. The lobbyist is trying to persuade the congressperson that if the oil industry is granted certain tax breaks provided by pending legislation, then benefits for all voters will result. Sketch a Bayes' diagram that represents, in a qualitative way, the relationship between tax breaks for the oil industry T and benefits for all voters B according to the lobbyist. The congressperson, however, doesn't buy the lobbyist's argument. She thinks that whether the voters benefit or not is completely independent of the passage of the pending legislation. In other words, she thinks that B is independent of T. Sketch a Bayes' diagram, also qualitatively, that represents this point of view.





SMOKING AND LUNG CANCER. A survey of the type taken in the early 1980s sought to determine the relationship between smoking and lung cancer. The study broke the 1,000,000 individuals who were surveyed into four groups: smokers who live in an urban area, smokers who live in a rural area, non-smokers who live in an urban area, and non-smokers who live in a rural area. Let symbols be defined as follws.

U: an individual from an urban area	$\bar{U}$: an individual from a rural area
S: an individual who smoked	$\bar{S}$: an individual who is a non-smoker
L: an individual who died of lung cancer	

Let given quantities be as follows.

no.( ): the number of the quantity within the parentheses

no.(U) = 700,000	no.(S) = 170,000	no.(SU) = 140,000
no.($\bar{U}$) = 300,000	no.($\bar{S}$) = 830,000	no.($S\bar{U}$) = 30,000

Let the probabilities for dying of lung cancer for each of the four target groups be as follows.

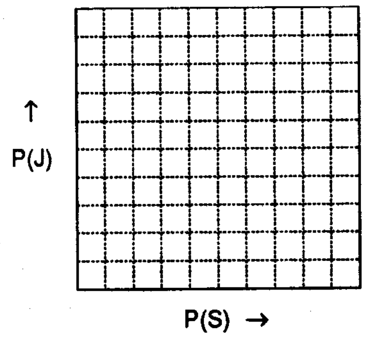
$P(L|S\underline{U}) = 0.000\ 85$
$P(L|\underline{S}\bar{U}) = 0.000\ 65$
$P(L|\underline{\bar{S}}\underline{U}) = 0.000\ 15$
$P(L|\bar{S}\bar{U}) = 0.000\ 01$

Find the following quantities, the first two of which have already been given.

$no.(S\underline{U})$	$no.(LS\underline{U})$	$no.(L\underline{U})$	$P(L\|\underline{U})$	$P(U\|L)$
$no.(S\bar{U})$	$no.(LS\bar{U})$	$no.(L\bar{U})$	$P(L\|\bar{U})$	$P(S\|L)$
$no.(\underline{\bar{S}}\underline{U})$	$no.(L\bar{S}U)$	$no.(L\underline{S})$	$P(L\|\underline{S})$	
$no.(\bar{S}\bar{U})$	$no.(L\bar{S}\bar{U})$	$no.(L\bar{S})$	$P(L\|\bar{S})$	

What are the major conclusions that can be drawn from this information?

***PART TIME JOBS.** In a small college 1000 students are from out of state and 3000 are state residents. One-hundred of the out-of-state students hold part-time jobs (20 hrs/wk or more) and 1500 of the in-state students have part-time jobs to help pay for textbooks, tuition, fees, and miscellaneous expenses. If a student is selected randomly from the 4000 students at the school, what is the probability for selecting a student who is both an out-of-state student and one who has a part-time job? Let S represent an in-state student and J a part-time job. Also, find the probability that a student who has a part-time job will be from out of state. Finally, construct a Bayes' diagram to describe the relationship between P(J) and P(S). What relationship in plausible reasoning does this represent?



***EINSTEIN'S PREDICTIONS.** Einstein's general theory of relativity is a theory of gravitation. It recognizes that the force of gravity cannot be distinguished from the effects produced by an acceleration through space. For example, when in an elevator accelerating upward you feel much heavier. This effect is the same as if the mass of the earth were to have been suddenly increased thereby exerting a greater gravitational pull upon your body. Not only did Einstein formulate the theory, but in addition he proposed three experiments that might be conducted that would either support the theory or refute it. One experiment had to do with the bending of starlight as it passes close to the sun on its way to earth. A second effect predicted by the theory was the reddening of light emitted by an extremely massive star as that light is attracted in a direction back toward the star while leaving that star's vicinity. A third prediction involved the orbital motion of the planet Mercury, which, being so close to the massive sun, is influenced by the sun's gravitational attraction. All three of these consequences C_1, C_2, and C_3 were eventually confirmed. The problem here is to assess changes in the probability that Einstein's theory is correct, P(T), as each of these predictions (consequences) is confirmed. Assume numerical values as necessary to sketch the three Bayes' diagrams with axes as shown below that describe the successive confirmation of the three consequences. Show that the posterior $P(T|C_1)$ is the prior for the second graph, and that the posterior of the second graph $P(T|C_1.C_2)$ is the prior for the third graph whose posterior is $P(T|C_1.C_2.C_3)$.